The Curved Motion of Bodies

Centripetal and Centrifugal Forces

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October 2015

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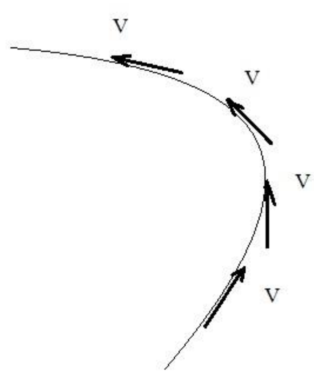
The curved motion of bodies has been studied since the time of Christiaan Huygens who extended from Newton's second law to derive the quadratic form of a force acting on an object on a circular path, or $F_c = mv^2/r$. At the time, this helped the study of orbits in the field of Astronomy in which philosophers were just as intrigued over Kepler's laws, and curved motion was considered something that primarily took place in the study of the motion of planets. It was most likely implied that anything that moved similar to the planets, whether it were through the air or on the ground, could also be handled by this same formulation. Some brief checking probably verified this, and since the planets remained nicely in their orbits, there appeared nothing else to worry about regarding how other bodies undergo this type of motion.

Centuries later, when transportation evolved yielding trains, cars, and planes, the objects that are affected by curved motion have become more clearly defined. We now know that cars and trains cannot take a turn too fast or they meet devastating consequences. But since derailment of trains and overturning vehicles are still being reported today, it implies that the study of curved motion of vehicles may yet to be finished.

The concept is that while speed is constant about a curve its direction will be changing, and since a velocity vector involves both a scalar magnitude and a direction, the change of direction itself constitutes an acceleration:



A velocity vector about a curved path has constant speed but keeps changing direction causing an acceleration.

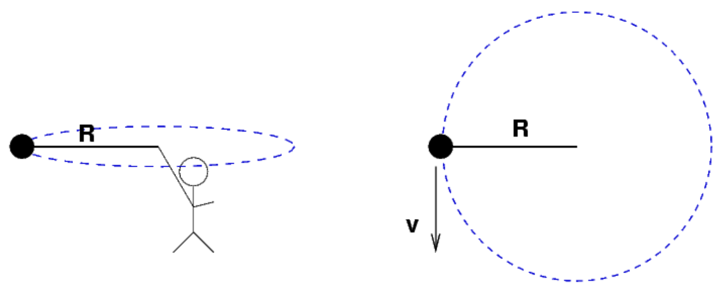
This observation was made by Isaac Newton and mathematically described by Christiaan Huygens in 1659,

$$F_c = mv^2/r$$

and states a centrally directed force F_c orthogonal to the object in motion on a curved path is equal to the mass *m* times the velocity *v* squared divided by the radius of the arc of motion *r*.

A body can also experience acceleration in the tangential direction and therefore possess an additional acceleration vector with the velocity vector, but will not always fit actual circumstances. Although a tangential component of acceleration perhaps provides a more complete – or more involved - dynamics problem, the actual circumstances of the real world persuades our discussion to focus on the constant speed about a curved path.

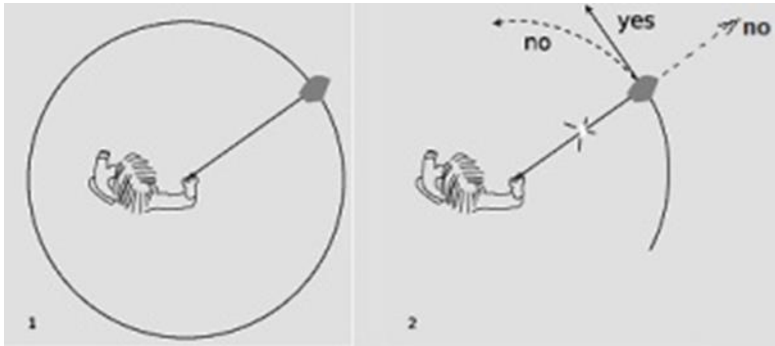
The simplest example most often referred to is that of a whirling object on the end of a string, such as that of a whirling ball:



side view　　　top view

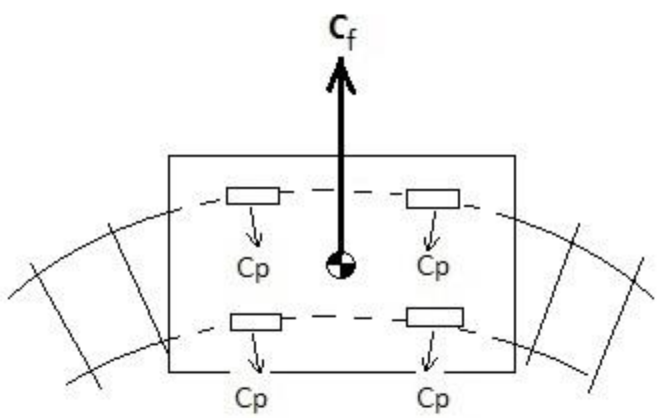
It is found that to make the ball whirl in a circular path at a constant speed one must always be "pulling" on the string. Therefore a force along the string is needed directed inward toward the center, called the Centripetal Force (or "center-seeking"). The ball itself is tending outward such that an equal and opposite force must also be acting on the ball, which is simply the application of Newton's Third Law, and is called Centrifugal Force (or "center-fleeing")

It was observed that if the string breaks in the above example, the ball does not travel outward as one might intuitively think - but in a straight line tangential to the curved path at the point where the string breaks. Because of this, some have begun to reason that centrifugal force was something of a "fictitious force" since it did not have the effect of "following-through" after the ball let go. But neither did the ball travel inward (even after all that pulling). Should not centripetal force also be fictitious ? Confusion stirs at this point but can easily be resolved if one realizes that when the string breaks forces are no longer transmitted. Therefore, the applied centripetal force and the reacting centrifugal force both *instantly become zero* when the string breaks. It can also be reasoned that breaking the string imposes a new condition on the problem, and therefore a new problem should be redefined (without the orthogonal forces, of course).



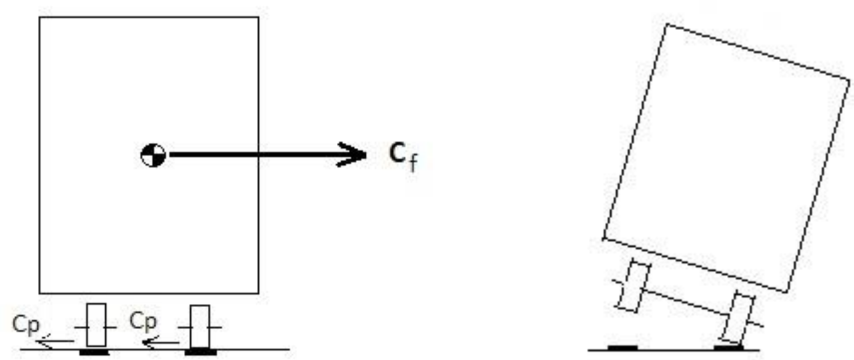
The important thing to realize is that while the string is intact, there *are* forces acting on the string *and* reacting on the ball. The centripetal and centrifugal forces are always present when the string is intact. Centripetal is the applied force of pulling the string inward to maintain the circular motion, and the centrifugal force is the outward reacting force causing the ball to tend outward. Both are present and both are real.

Extending this idea to a train travelling around a curved track, as the wheels are steered by the curved track, they will apply a centripetal force at the contact interface between the wheel-and-track. This is the inward "pulling" force toward the center of motion. The train itself will react outwardly, however, with a centrifugal force acting through the center of gravity of the train car.



As a train car is steered by the curved track, centripetal forces will be applied at the wheel-track interface and will sum to equal the Centrifugal Force reacting in opposite direction through the center-of-gravity of the train car.

If the speed is too great, the forces given by the relation $F_c = mv^2/r$ will cause a *couple* or overturning moment tipping the train off the tracks. When the train breaks contact with the tracks, it is similar to when the string breaks in the case of the whirling ball. The centripetal and centrifugal forces become zero. This is quite obvious for the centripetal force since the wheel has lifted off the track and can no longer transmit a force. If the centripetal force has become zero then the equal and opposite centrifugal force is also zero. But the damage has been done at this point. The train has derailed and wants to travel in a straight line off the tracks.



When the applied centripetal forces become great enough, the reacting Centrifugal Force will tip the train over, breaking contact with the track. At this point, the forces become zero and the path of the train will continue in a tangential straight-line off the tracks. (Weight of the train car not shown)

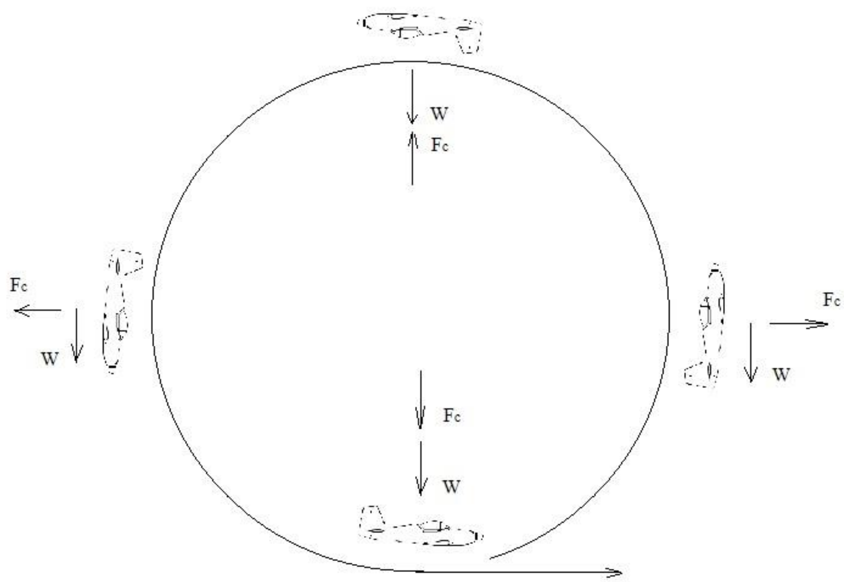
It should be noted that the tipping tendency is caused by the centrifugal force acting through the center of gravity which couples with the centripetal forces at the wheel-track interface. If both forces were aligned, as in the whirling ball, this coupling moment would not exist. It is the orientation of forces that causes a coupling phenomenon, but this warning is quite routinely known in the field of engineering. As in the whirling ball example, the centripetal and centrifugal forces exist and are real while curved motion is maintained, but instantly become zero when the medium through which the forces are transmitted become severed.

A question posed is that if one side of the train car separates from the track, that is, one set of wheels lifts off the track and possibly balances, can the train car fall back onto the tracks ? As the train is tipped onto one set of wheels while the train is persistently on a curved path, the centrifugal force will remain relentless and with full force continue to tip the train completely off the tracks until there is no contact whatsoever, and the train derails. Also, the entire sequence will not occur in stages, but all at once. The only possibility is if the track begins to straighten at the instant at which the train begins to tip, thus alleviating the centrifugal force and allowing the train to fall back onto the tracks. This occurrence would be quite rare due to the exact timing required, and therefore the most probable condition is that when tipping is first pending, and wheel-to-track contact is first broken, the train will proceed in a sequence to derail with no chance to correct itself. The only other alternative is if braking were applied at the exact moment of pending-tipping which could also alleviate the centrifugal force. This occurrence would also be quite rare, with braking needed at the exact moment of which there would be no warning to apply such action.

The critical tipping moment of a train car caused by a force through the C.G. (center of gravity) can be obtained by standard calculation, but also verified by laboratory simulation. The critical tipping moment caused by the centrifugal force is the basis in determining the maximum allowable speed of a train car travelling on a curved track.

The above follows for vehicles that use tire traction in making turns. The mechanical force of *friction* is the applied centripetal force at the tire-road interface, and the outward reacting force through the center of gravity of the vehicle is the centrifugal force. Since a person operating a vehicle has variable control of its steering (unlike a fixed train track) if tipping is pending and sensed by the driver, the wheels can be straightened out almost immediately to alleviate a disaster. This will depend on the reflex-sensitivity of the driver, but steering is thought to be one of the more finely tuned controls on today's cars, mostly due to power steering.

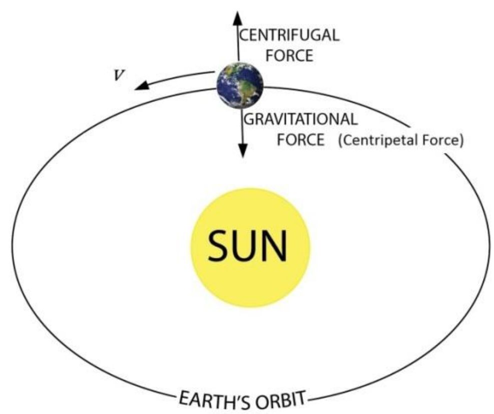
Aircraft performing maneuvers can be subjected to centrifugal forces. One of these is the loop maneuver possibly performed by world war aircraft, but now days performed during air shows to demonstrate an aircraft's flight maneuverability. Aerobatic aircraft usually have wings that are symmetric through their cross-section and generate lift by using angle of attack. A centripetal force is applied through the plane's elevator to begin the loop, and adjusted by way of flight controls to maintain a consistent loop pattern while encountering the force of gravity when undergoing an inverted orientation.



The Centrifugal Force F_c will counteract the weight W of the inverted aircraft at the top of the loop but will compound with the weight nearing the loop bottom causing a 2G downward force just before exiting the loop maneuver. (Forces shown not necessarily to scale. Lift and elevator forces not shown).

For a circular loop, a constant centrifugal force can be felt by the plane throughout the entire loop and acts through the center of gravity. At the top of the loop when the aircraft is inverted, F_c will counteract the weight W, "hanging" in a state of equilibrium. Nearing the bottom of the loop, however, the centrifugal force will compound or add to the weight W causing the highest downward force on the aircraft during the entire loop. The last quadrant of the loop may be cautioned to pilots as critical for deciding a safe altitude, and after passing through the last ninety-degrees of arc, F_c should completely go to zero allowing the pilot to confidently resume a straight-and-level flying position. If a pilot chooses to make a perfectly circular loop, as shown above, the centrifugal force F_c can be maintained constant and calculated to counteract the weight of the aircraft at the top of the loop, and also prepare for a 2G downward force at the bottom. Any lift forces underneath the wing, and the elevator force pitching the aircraft relatively upward or downward, are not shown above and the forces shown are not necessarily to scale and may only represent a sum resultant when combined with the lift and elevator forces. All forces must be considered when calculating the net centrifugal force needed to perform the loop maneuver, and to achieve a constant centrifugal force, a constant speed is maintained along with a perfectly circular loop.

For planetary motion, the Centrifugal force is what holds the planets up and prevents them from falling into the Sun. Gravity provides the Centripetal force thought to be the "driving force" to cause its circular orbit. Orbital motion can only take place with both forces co-existing. Without centripetal force, there would be nothing to "cause" the circular motion. Without centrifugal force, the planets would crash into the Sun. Both are present and both are real.



To maintain the Earth's orbit about the Sun, the Centrifugal Force must coexist with the Centripetal Force - which in the case of orbiting bodies has the name Gravitational Force.

Upon surveying the extent of applications involving curved motion, it presently appears to narrow down to only a handful of several applications; instead of faced with a multitude of potential examples which must have perplexed Huygens and Newton. Under those circumstances, where no one knows for certain how far a concept will extend, only the most general offering of a formula can be presented for its time era. However, the very same formula that was derived over 300 years ago is still in existence today to define the magnitude of forces for curved motion. But although it is the same formula from an old era, it does not mean it is without newly attached findings. Now days, we know curved motion exists for only specific but rather important cases, which are the transportation-related issues involving trains, cars, and boats; planes performing aerobatic maneuvers; and a satisfactory understanding of satellites, both natural and artificial.